

10. (a) When the small sphere is released at the edge of the large “bowl” (the hemisphere of radius R), its center of mass is at the same height at that edge, but when it is at the bottom of the “bowl” its center of mass is a distance r above the the bottom surface of the hemisphere. Since the small sphere descends by $R - r$, its loss in gravitational potential energy is $mg(R - r)$, which, by conservation of mechanical energy, is equal to its kinetic energy at the bottom of the track.
- (b) Using Eq. 12-5 for K , the asked-for fraction becomes

$$\frac{K_{\text{rot}}}{K} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}I\omega^2 + \frac{1}{2}Mv_{\text{com}}^2} = \frac{1}{1 + \left(\frac{M}{I}\right) \left(\frac{v_{\text{com}}}{\omega}\right)^2} .$$

Substituting $v_{\text{com}} = R\omega$ (Eq. 12-2) and $I = \frac{2}{5}MR^2$ (Table 11-2(f)), we obtain

$$\frac{K_{\text{rot}}}{K} = \frac{1}{1 + \left(\frac{5}{2R^2}\right) R^2} = \frac{2}{7} .$$

- (c) The small sphere is executing circular motion so that when it reaches the bottom, it experiences a radial acceleration upward (in the direction of the normal force which the “bowl” exerts on it). From Newton’s second law along the vertical axis, the normal force N satisfies $N - mg = ma_{\text{com}}$ where $a_{\text{com}} = v_{\text{com}}^2/(R - r)$. Therefore,

$$N = mg + \frac{mv_{\text{com}}^2}{R - r} = \frac{mg(R - r) + mv_{\text{com}}^2}{R - r} .$$

But from part (a), $mg(R - r) = K$, and from Eq. 12-5, $\frac{1}{2}mv_{\text{com}}^2 = K - K_{\text{rot}}$. Thus,

$$N = \frac{K + 2(K - K_{\text{rot}})}{R - r} = 3\frac{K}{R - r} - 2\frac{K_{\text{rot}}}{R - r} .$$

We now plug in $R - r = K/mg$ and use the result of part (b):

$$N = 3mg - 2mg \left(\frac{2}{7}\right) = \frac{17}{7} mg .$$